Landau Tmp/09/97.
September 1997

MIRROR SYMMETRY AS A POISSON-LIE T-DUALITY.

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Abstract

The transformation properties of the $N=2$ Virasoro superalgebra generators under Poisson-Lie T-duality in (2,2)-superconformal WZNW and Kazama-Suzuki models is considered. It is shown that Poisson-Lie T-duality acts on the $N=2$ super-Virasoro algebra generators as a mirror symmetry does: it unchanges the generators from one of the chirality sectors while in another chirality sector it changes the sign of $U(1)$ current and interchanges spin-3/2 currents. We discuss Kazama-Suzuki models generalization of this transformation and show that Poisson-Lie T-duality acts as a mirror symmetry also.

PACS: 11.25Hf; 11.25 Pm.

Keywords: Strings, Duality, Superconformal Field Theory.

Introduction.

The $N=2$ superconformal field theories (SCFT's) play an important role in diverse aspects of superstring theory. As the first application in the superstrings the $N=2$ SCFT's was appeared in Gepner's superstring vacua construction [1] (see also [2]), where it was shown that the $N=2$ SCFT's may describe Calabi-Yau manifolds compactifications of the superstrings. Since then the $N=2$ SCFT's, and the profound structures assotiated with them are an area of investigation. On the other hand, it is well known that the string vacua, considered as conformal field thoeries, in general, have deformations under which the geometry of the target space changes. These deformations include the discrete duality transformations, so called T-duality, which are symmetries of the underlying conformal field theory [3], [4].

Mirror symmetry [5] discovered in superstring theory is the special type of T-duality. At the level of conformal field theory, it could be formulated as an isomorphism between two theories, amounting to a change of sign of the $U(1)$ generator and interchange spin-3/2

generators of the left- moving (or right-moving) $N = 2$ superconformal algebra.

Mirror symmetry has mostly been studied in the context of Calabi-Yau superstring compactifications. Though this approach is quite general, since it allows for the moduli of the theory to be varied, its quantization seems to be fuzzy because it is not known what is the quantum Calabi-Yau σ-model. In fact the only rigorously established example of mirror symmetry, the Green-Plesser construction [6], is based on the tensor products of the $N = 2$ minimal models [7].

In this note we propose algebraic approach to the mirror symmetry based on Poisson-Lie (PL) T-duality. The Poisson-Lie T-duality, recently discoverd by C. Klimcik and P. Severa [8] is a generalization of the standard non-Abelian T-duality [9]- [13]. This generalized duality is associated with two groups forming a Drinfeld double [14] and the duality transformation exchanges their roles. This approach has recieved futher developments in the series of works [15]- [19].

The supersymmetric generalization of PL T-duality was considered in [20]- [24]. In particular, in [24] it was shown that PL T-duality in $N = 2$ SWZNW models is governed by the complex Heisenberg doubles associated with the group manifolds of the models and PL T-duality mappings are given on-shell by the special super Kac-Moody gauge transformations thus establishing (on-shell) PL self-duality of the $N = 2$ SWZNW models.

In the recent paper of Klimcik [23] (1,1) supersymmetric PL T-duality was formulated off-shell. It proves PL T-duality in $N = 2$ SWZNW models.

The present note is based on the results of [24] (so it can be considered as its appendix) and devoted to the investigation (on-shell) of the transformation properties of the $N = 2$ super-Virasoro algebra under the PL T-duality in the $N = 2$ SWZNW and Kazama-Suzuki models [25]. We will show that PL T-duality transforms the generators of the $N = 2$ super- Virasoro algebra exactly the same way as the mirror duality does. Thus we obtain more rigorous arguments for the conjecture proposed in [26] that mirror symmetry can be related to a gauge symmetry of the self-dual points of the $N = 2$ SCFT's moduli space (the N=0 version of this conjecture see in [27]).

In the section 1 we briefly review the results of paper [24]. In section 2 we describe transformation properties of N=1 super-Kac-Moody algebra currents under PL T-duality. Then we describe in section 3 PL T-duality transformations of the left-moving and right-moving $N = 2$ super-Virasoro algebras in the $N = 2$ SWZNW and Kazama-Suzuki models.

1. The classical $N = 2$ superconformal WZNW model.

In this section we briefly review the $N = 2$ SWZNW models following [28, 29, 24].

We parametrize super world-sheet introducing the light cone coordinates $x_{\pm}$, and grassman coordinates $\Theta_{\pm}$ (we shall use N=1 superfield formalism). The generators of supersymmetry and covariant derivatives are

$$Q_{\mp} = \frac{\partial}{\partial\Theta_{\pm}} + \imath\Theta_{\pm}\partial_{\mp},\ D_{\mp} = \frac{\partial}{\partial\Theta_{\pm}} - \imath\Theta_{\pm}\partial_{\mp}. \quad (1)$$

They satisfy the relations

$$\{D_{\pm}, D_{\pm}\} = -\{Q_{\pm}, Q_{\pm}\} = -\imath 2\partial_{\pm},\ \{D_{\pm}, D_{\mp}\} = \{Q_{\pm}, Q_{\mp}\} = \{Q, D\} = 0, \quad (2)$$

where the brackets $\{,\}$ denote the anticommutator. The superfield of N=1 SWZNW model

$$G = g + \imath\Theta_{-}\psi_{+} + \imath\Theta_{+}\psi_{-} + \imath\Theta_{-}\Theta_{+}F \quad (3)$$

takes values in a real Lie group $\mathbf{G}$. We will assume that its Lie algebra $\mathbf{g}$ is endowed with ad-invariant nondegenerate inner product $<,>$.

The inverse group element G^{-1} is defined from the relation

$$G^{-1}G = 1 \tag{4}$$

and has the decomposition

$$G^{-1} = g^{-1} - \imath\Theta_- g^{-1}\psi_+ g^{-1} - \imath\Theta_+ g^{-1}\psi_- g^{-1} - \imath\Theta_-\Theta_+ g^{-1}(F + \psi_- g^{-1}\psi_+ - \psi_+ g^{-1}\psi_-)g^{-1} \tag{5}$$

The action of N=1 SWZNW model is given by

$$\begin{aligned} S_{swz} = \int d^2x d^2\Theta (< G^{-1}D_+G, G^{-1}D_-G >) \\ - \int d^2x d^2\Theta dt < G^{-1}\frac{\partial G}{\partial t}, \{G^{-1}D_-G, G^{-1}D_+G\} > . \end{aligned} \tag{6}$$

The action (6) is invariant under the super-Kac-Moody

$$\begin{aligned} \delta_{a_+}G(x_+, x_-, \Theta_+, \Theta_-) = a_+(x_-, \Theta_+)G(x_+, x_-, \Theta_+, \Theta_-), \\ \delta_{a_-}G(x_+, x_-, \Theta_+, \Theta_-) = -G(x_+, x_-, \Theta_+, \Theta_-)a_-(x_+, \Theta_-), \end{aligned} \tag{7}$$

where $a_\pm$ are $\mathbf{g}$-valued superfields and N=1 supersymmetry transformations [28]

$$\begin{aligned} G^{-1}\delta_{\epsilon_+}G = (G^{-1}\epsilon_+ Q_+ G), \\ \delta_{\epsilon_-}GG^{-1} = \epsilon_- Q_- GG^{-1}. \end{aligned} \tag{8}$$

The action of $N = 2$ SWZNW model is given by (6) also. An additional ingredient demanded by the $N = 2$ Virasoro superalgebra of symmetries is a complex structure on the finite-dimensional Lie algebra of the model which is skew-symmetric with respect to the inner product $<,>$ [30, 31, 29]. By the definition J is a complex structure on the Lie algebra $\mathbf{g}$ if it is a complex structure on the vector space $\mathbf{g}$ which satisfies the equation

$$[Jx, Jy] - J[Jx, y] - J[x, Jy] = [x, y] \tag{9}$$

for any elements x, y from $\mathbf{g}$. It is clear that the corresponding Lie group is a complex manifold with left (or right) invariant complex structure. In the following we shall denote the real Lie group and the real Lie algebra with the complex structure satisfying (9) as the pairs $(\mathbf{G}, J)$ and $(\mathbf{g}, J)$ correspondingly.

If the complex structure J on the Lie algebra is fixed then it defines the second supersymmetry transformation [29]

$$\begin{aligned} (G^{-1}\delta_{\eta_+}G)^a = \eta_+(J_l)^a_b(G^{-1}D_+G)^b, \\ (\delta_{\eta_-}GG^{-1})^a = \eta_-(J_r)^a_b(D_-GG^{-1})^b, \end{aligned} \tag{10}$$

where J_l, J_r are the left invariant and right invariant complex structures on $\mathbf{G}$ which correspond to the complex structure J.

The Sugawara construction of the $N = 2$ Virasoro superalgebra generators was given in [30, 31, 29, 32].

The notion of Manin triple closely related with the complex structure on the Lie algebra. By the definition [14], a Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ consists of a Lie algebra $\mathbf{g}$, with nondegenerate invariant inner product $<,>$ and isotropic Lie subalgebras $\mathbf{g}_\pm$ such that $\mathbf{g} = \mathbf{g}_+ \oplus \mathbf{g}_-$ as a vector space.

Suppose the existence of the nondegenerate invariant inner product $<,>$ on $(\mathbf{g}, J)$ so that the complex structure J is skew-symmetric with respect to $<,>$ and consider the complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$. Let $\mathbf{g}_\pm$ be $\pm\imath$ eigenspaces of J in the algebra $\mathbf{g}^{\mathbb{C}}$ then $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_+, \mathbf{g}_-)$ is a complex Manin triple . Moreover it can be proved that there exists the one-to-one correspondence between the complex Manin triple endowed with antilinear involution which conjugates isotropic subalgebras $\tau : \mathbf{g}_\pm \rightarrow \mathbf{g}_\mp$ and the real Lie algebra endowed with ad-invariant nondegenerate inner product $<,>$ and the complex structure J which is skew-symmetric with respect to $<,>$ [30]. The conjugation can be used to extract the real form from the complex Manin triple.

In this note we concentrate on the $N = 2$ SWZNW models on the compact groups (the extension on the noncompact groups is straightforward) that is we shall consider complex Manin triples such that the corresponding antilinaer involutions will coincide with the hermitian conjugations. Hence it will be implied in the following $\mathbf{G}$ is a subgroup in the group of unitary matrices and the matrix elements of the superfield G satisfy the relations:

$$\bar{g}^{mn} = (g^{-1})^{nm},\ \bar{\psi}^{mn}_\pm = (\psi^{-1})^{nm}_\pm,\ \bar{F}^{mn} = (F^{-1})^{nm}, \tag{11}$$

where we have used the following notations

$$\psi^{-1}_\pm = -g^{-1}\psi_\pm g^{-1},\ F^{-1} = -g^{-1}(F + \psi_- g^{-1}\psi_+ - \psi_+ g^{-1}\psi_-)g^{-1}. \tag{12}$$

Now we have to consider some geometric properties of the $N = 2$ SWZNW models closely related with the existence of the complex structures on the groups.

Let's fix some compact Lie group with the left invariant complex structure $(\mathbf{G}, J)$ and consider its Lie algebra with the complex structure $(\mathbf{g}, J)$. The complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$ has the Manin triple structure $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_+, \mathbf{g}_-)$. The Lie group version of this triple is the double Lie group $(\mathbf{G}^{\mathbb{C}}, \mathbf{G}_+, \mathbf{G}_-)$ [33, 34, 35], where the exponential subgroups $\mathbf{G}_\pm$ correspond to the Lie algebras $\mathbf{g}_\pm$. The real Lie group $\mathbf{G}$ is extracted from its complexification with help of the hermitian conjugation τ

$$\mathbf{G} = \{g \in \mathbf{G}^{\mathbb{C}} | \tau(g) = g^{-1}\} \tag{13}$$

Each element $g \in \mathbf{G}^{\mathbb{C}}$ from the vicinity $\mathbf{G_1}$ of the unit element from $\mathbf{G}^{\mathbb{C}}$ admits two decompositions

$$g = g_+ g_-^{-1} = \tilde{g}_- \tilde{g}_+^{-1}, \tag{14}$$

where $\tilde{g}_\pm$ are dressing transformed elements of $g_\pm$ [35]:

$$\tilde{g}_\pm = (g_\pm^{-1})^{g_\mp} \tag{15}$$

Taking into account (13) and (14) we conclude that the element g ($g \in \mathbf{G_1}$) belongs to $\mathbf{G}$ iff

$$\tau(g_\pm) = \tilde{g}_\mp^{-1} \tag{16}$$

These equations mean that we can parametrize the elements from

$$\mathbf{C_1} \equiv \mathbf{G_1} \cap \mathbf{G} \tag{17}$$

by the elements from the complex group $\mathbf{G}_+$ (or $\mathbf{G}_-$), i.e. we can introduce complex coordinates (they are just matrix elements of g_+ (or g_-)) in the strat $\mathbf{C_1}$. To do it one needs to solve with respect to g_- the equation:

$$\tau(g_-) = (g_+)^{g_-^{-1}} \tag{18}$$

(to introduce $\mathbf{G}_-$-coordinates on $\mathbf{G_1}$ one needs to solve with respect to g_+ the equation

$$\tau(g_+) = (g_-)^{g_+^{-1}}). \tag{19}$$

Thus the formulas (14), (18) ((19)) define the mapping

$$\phi_1^+ : \mathbf{G}_+ \to \mathbf{C_1} \tag{20}$$

$$(\phi_1^- : \mathbf{G}_- \to \mathbf{C_1}) \tag{21}$$

For the $N = 2$ SWZNW model on the group $\mathbf{G}$ we obtain from (14) the decompositions for the superfield (4) (which takes values in $\mathbf{C_1}$)

$$G(x_+, x_-) = G_+(x_+, x_-)G_-^{-1}(x_+, x_-) = \tilde{G}_-(x_+, x_-)\tilde{G}_+^{-1}(x_+, x_-) \tag{22}$$

To generalize (14), (16) one have to consider the set W (which we shall assume in the following to be discret and finite set) of classes $\mathbf{G}_+\backslash\mathbf{G}^{\mathbb{C}}/\mathbf{G}_-$ and pick up a representative w for each class $[w] \in W$. It gives us the stratification of $\mathbf{G}^{\mathbb{C}}$ [34]:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W} \mathbf{G}_+ w \mathbf{G}_- = \bigcup_{[w]\in W} \mathbf{G_w} \tag{23}$$

There is the second stratification:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W} \mathbf{G}_- w \mathbf{G}_+ = \bigcup_{[w]\in W} \mathbf{G^w} \tag{24}$$

We shall assume, in the following, that the representatives w have picked up to satisfy the unitarity condition:

$$\tau(w) = w^{-1} \tag{25}$$

It allows us to generalize (14), (16) as follows

$$g = w g_+ g_-^{-1} = w \tilde{g}_- \tilde{g}_+^{-1}, \tag{26}$$

where

$$g_+ \in \mathbf{G}_+^{\mathbf{w}},\ \tilde{g}_- \in \mathbf{G}_-^{\mathbf{w}} \tag{27}$$

and

$$\mathbf{G}_+^{\mathbf{w}} = \mathbf{G}_+ \cap w^{-1}\mathbf{G}_+ w,\ \mathbf{G}_-^{\mathbf{w}} = \mathbf{G}_- \cap w^{-1}\mathbf{G}_- w. \tag{28}$$

It is clear that there exists also an appropriate generalization of (22) for the decompositions (26).

In order to the element g belongs to the real group $\mathbf{G}$ the elements $g_{\pm}, \tilde{g}_{\pm}$ from (26) should satisfy (16). Thus the formulas (26, 27), (18) ((19)) define the mapping

$$\phi_w^+ : \mathbf{G}_+^{\mathbf{w}} \to \mathbf{C}_{\mathbf{w}} \equiv \mathbf{G}_{\mathbf{w}} \cap \mathbf{G} \tag{29}$$

$$(\phi_w^- : \mathbf{G}_-^{\mathbf{w}} \to \mathbf{C}_{\mathbf{w}} \equiv \mathbf{G}_{\mathbf{w}} \cap \mathbf{G}). \tag{30}$$

To formulate the main result of the paper [24] one needs to introduce some notations. Let

$$\{R_i, i = 1, ..., d\}, \tag{31}$$

be the basis in the Lie subalgebra $\mathbf{g}_+$ and

$$\{R^i, i = 1, ..., d\}, \tag{32}$$

be the basis in the Lie subalgebra $\mathbf{g}_-$ so that (31, 32) constitute the orthonormal basis in $\mathbf{g}^{\mathbb{C}}$:

$$< R^i, R_j >= \delta^i_j. \tag{33}$$

We identify the Lie algebra $\mathbf{g}^{\mathbb{C}}$ with the space of complex left invariant vector fields on the group $\mathbf{G}$. To each decomposition (22) or its generalization for the mappings into others strats $\mathbf{C}_{\mathbf{w}}$ we introduce the superfields

$$\begin{aligned} \rho^+ = G_+^{-1} D G_+ = \rho^i R_i,\ \rho^- = G_-^{-1} D G_- = \rho_i R^i, \\ \tilde{\rho}^+ = \tilde{G}_+^{-1} D \tilde{G}_+ = \tilde{\rho}^i R_i,\ \tilde{\rho}^- = \tilde{G}_-^{-1} D \tilde{G}_- = \tilde{\rho}_i R^i. \end{aligned} \tag{34}$$

These superfields correspond to the left invariant 1-forms on $\mathbf{G}_{\pm}$

$$r^{\pm} = g_{\pm}^{-1} d g_{\pm},\ \tilde{r}^{\pm} = \tilde{g}_{\pm}^{-1} d \tilde{g}_{\pm}. \tag{35}$$

In [24] the following statements was proved:

- the mappings (29) are holomorphic and define the natural action of the complex group $\mathbf{G}_+$ on $\mathbf{G}$ generated by the holomorphic vector fields $\{S_i, i = 1, ..., d\}$; the set W parametrizes $\mathbf{G}_+$-orbits $\mathbf{C}_{\mathbf{w}}$.
- The Lagrangian Λ of the model is given by

$$\Lambda = \frac{\imath}{2} \Omega_{cb} J^c_a \rho^a_+ \rho^b_-, \tag{36}$$

  where Ω_{cb} is Semenov-Tian-Shansky symplectic form on $\mathbf{G}$ [33], J^c_a is left invariant complex structure on $\mathbf{G}$ and we have used common notation $\rho^a, a = 1, ..., 2d$ for the 1-forms $\rho^i, \bar{\rho}^i$. Remark that stratifications (23), (24) code degegnerations of Ω [34] thus the formula (36) is true within each strat $\mathbf{C}_{\mathbf{w}}$.

- $(\mathbf{G}, J)$-SWZNW model admits PL symmetry with respect to the holomorphic $\mathbf{G}_+$-action, i.e. the following conditions are satisfied on the extremals

$$L_{S_i}\Lambda = f_i^{jk}(A_+)_j(A_-)_k$$
$$L_{\bar{S}_i}\Lambda = \bar{f}_i^{jk}(A_+)_{\bar{j}}(A_-)_{\bar{k}}, \tag{37}$$

where $L_{S_i}, L_{\bar{S}_i}$ mean the Lie derivatives along the vector fields $S_i, \bar{S}_i$ ($\bar{S}_i$ are complex conjugated to S_i), f_j^{ik} are the structure constants of the Lie algebra $\mathbf{g}_-$ ($\bar{f}_i^{jk}$ are complex conjugated to f_j^{ik}) and the Noether currents $A_i, A_{\bar{i}}$ are given by

$$(A_-)_i = (\rho_-)_i, \ (A_+)_i = \imath(J\rho_+)_i, \ (A_\pm)_{\bar{i}} = (\bar{A}_\pm)_i. \tag{38}$$

The equations (37) are equivalent to zero curvature equations for the F_{+-}-component of the super stress tensor F_{MN} [8, 24]

$$(F_{+-})_i \equiv D_+(A_-)_i + D_-(A_+)_i - f_i^{nm}(A_+)_n(A_-)_m = 0$$
$$(F_{+-})_{\bar{i}} \equiv D_+(A_-)_{\bar{i}} + D_-(A_+)_{\bar{i}} - \bar{f}_i^{nm}(A_+)_{\bar{n}}(A_-)_{\bar{m}} = 0 \tag{39}$$

Using the standard arguments of the super Lax construction [37] one can show that from (39) it follows that the connection is flat

$$F_{MN} = 0, \ M, N = (+, -, +, -). \tag{40}$$

With the appropriate modifications the above statements are true also for the mappings (30) and $\mathbf{G}_-$-action on $\mathbf{G}$. Due to this observation PL self-duality $(\mathbf{G}, J)$- SWZNW models was proved in [24].

2. PL T-self-duality in the $N = 2$ SWZNW models.

In this section we consider PL T-duality in $(\mathbf{G}, J)$-SWZNW models and obtain transformation formulas of the super-Kac-Moody currents following to [8, 24].

Due to the equation (40) we may associate to each extremal surface $G_+(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}_+$, a mapping ("Noether charge") $V_-(x_+, x_-, \Theta_+, \Theta_-)$ from the super world-sheet into the group $\mathbf{G}_-$ such that

$$(A_\pm)_i = -(D_\pm V_- V_-^{-1})_i. \tag{41}$$

Now we build the following surface in the double $\mathbf{G}^{\mathbb{C}}$:

$$F(x_+, x_-, \Theta_+, \Theta_-) = G_+(x_+, x_-, \Theta_+, \Theta_-)V_-(x_+, x_-, \Theta_+, \Theta_-). \tag{42}$$

In view of (38,34) it is natural to represent V_- as the product

$$V_- = G_-^{-1}H_-^{-1} \tag{43}$$

, where G_- is determined from (18) and H_- satisfies the equation

$$D_-H_- = 0. \tag{44}$$

Therefore the surface (42) can be rewritten in the form

$$F(x_\pm, \Theta_\pm) = G(x_\pm, \Theta_\pm)H_-^{-1}(x_+, \Theta_-), \tag{45}$$

where $G(x_\pm,\Theta_\pm)\in\mathbf{G}$ is the solution of ($\mathbf{G}$)-SWZNW model. Now we represent the $\mathbf{G}_-$-valued field H_- in terms of conservation currents $I_+\equiv G^{-1}D_+G$ of the model. To do it let's denote by ξ the canonic $\mathbf{g}^{\mathbb{C}}$-valued left invariant 1-form on the group $\mathbf{G}$. It is obvious that

$$
\begin{aligned}
\xi &= \xi^j R_j + \xi_j R^j,\\
J\xi^k &= -\imath\xi^k,\ \ J\xi_k = \imath\xi_k.
\end{aligned}
\tag{46}
$$

Using the first decomposition from (14) we get

$$
\xi = g_- r^+ g_-^{-1} - l^-. \tag{47}
$$

Let's introduce the matrices

$$
\begin{aligned}
g_- R_i g_-^{-1} &= M_{ij}R^j + N_i^j R_j,\\
g_+ R^i g_+^{-1} &= P^{ij}R_j + Q^i_j R^j,\\
g_- R^i g_-^{-1} &= (N^*)^i_j R^j,\\
g_+ R_i g_+^{-1} &= (Q^*)^j_i R_j.
\end{aligned}
\tag{48}
$$

Using these matrices and (33) we can express the 1-forms r_i, in terms of ξ^i:

$$
r_i = ((N^*)^{-1})^j_i(-\xi_j + M_{nj}(N^{-1})^n_k\xi^k). \tag{49}
$$

Then

$$
\imath J r_i = ((N^*)^{-1})^j_i(\xi_j + M_{nj}(N^{-1})^n_k\xi^k). \tag{50}
$$

From the other hand in view of (43) we shall get

$$
(A_+)_i = (\rho_+)_i + (G_-^{-1}H_-^{-1}D_+H_-G_-)_i. \tag{51}
$$

Comparing (50) and (51) and taking into account (38), (48) we conclude

$$
H_-^{-1}D_+H_- = 2(I_+)^-, \tag{52}
$$

where $(I_+)^-$ is $\mathbf{g}_-$-projection of I_+. Though the formula (52) is obtained for the mappings into the strat $\mathbf{C_1}$ we claim that it remains to be true for the mappings into another strats, so that (45) is correct on the super world-sheet everywhere.

The solution and the "Noether charge" of the dual σ-model are given by "dual" parametrization of the surface (42) [8]

$$
F(x_+,x_-,\Theta_+,\Theta_-) = \breve{G}_-(x_+,x_-,\Theta_+,\Theta_-)\breve{V}_+(x_+,x_-,\Theta_+,\Theta_-), \tag{53}
$$

where $\breve{G}_-(x_+,x_-,\Theta_+,\Theta_-)\in\mathbf{G}_-$ and $\breve{V}_+(x_+,x_-,\Theta_+,\Theta_-)\in\mathbf{G}_+$. Thus in the dual σ-model Drinfeld's dual group to the group $\mathbf{G}_+$ should acts, i.e. it should be a σ-model on the orbits of the group $\mathbf{G}_-$ and with respect to this action the dual to (37) PL symmetry conditions should be satisfied:

$$
\begin{aligned}
L_{S^i}\breve{\Lambda} &= f^i_{jk}(\breve{A}_+)^j(\breve{A}_-)^k,\\
L_{\bar{S}^i}\breve{\Lambda} &= \bar{f}^i_{jk}(\breve{A}_+)^{\bar{j}}(\breve{A}_-)^{\bar{k}},
\end{aligned}
\tag{54}
$$

where $\{S^i, \bar{S}^i, i = 1, ..., d\}$ are the vector fields which generate the $\mathbf{G}_-$-action, $\breve{\Lambda}, \breve{A}^j_\pm, \breve{A}^{\bar{j}}_\pm$ are the Lagrangian and the Noether currents in the dual σ-model. It was argued in [24] $(\mathbf{G}, J)$-SWZNW models are PL self-dual models and (53) can be rewritten as follows

$$F(x_\pm, \Theta_\pm) = \breve{G}(x_\pm, \Theta_\pm) H_+^{-1}(x_+, \Theta_-), \tag{55}$$

where $\breve{G}(x_\pm, \Theta_\pm)$ is the dual solution of $\mathbf{G}$-SWZNW model. Similar to the equation (52) we can obtain

$$H_+^{-1} D_+ H_+ = 2(\breve{I}_+)^+, \tag{56}$$

where $(\breve{I}_+)^+$ is $\mathbf{g}_+$-projection of $\breve{I}_+ \equiv \breve{G}^{-1} D_+ \breve{G}$.

From (52,56) we conclude that under PL T-duality

$$t : G(x_\pm, \Theta_\pm) \to \breve{G}(x_\pm, \Theta_\pm) \tag{57}$$

the conserved current I_+ transforms as

$$t : (I_+)^- \to (\breve{I}_+)^+, \ (I_+)^+ \to (\breve{I}_+)^-. \tag{58}$$

Moreover as it follows from (45,55) the conserved currents $I_- \equiv D_- G G^{-1}$ map under PL T-duality identicaly:

$$t : (I_-)^\pm \to (\breve{I}_-)^\pm. \tag{59}$$

3. PL T-duality and Mirror symmetry.

In this section we consider the behaviour of the $N = 2$ super-Virasoro algebra generators under PL T-duality. Let's consider at first the transformation low for the components of the supercurrent I_+. On the extremals of $\mathbf{G}$-SWZNW model we have the following expansion of I_+

$$I_+ = \imath g^{-1}\psi_+ - \imath\Theta_-(g^{-1}\partial_+ g + \imath g^{-1}\psi_+ g^{-1}\psi_+). \tag{60}$$

We introduce the notations:

$$\begin{aligned} \phi_+ = g^{-1}\psi_+, \ q_+ = g^{-1}\partial_+ g, \\ j_+ = q_+ + \frac{\imath}{2}\{\phi_+, \phi_+\}. \end{aligned} \tag{61}$$

Then

$$I_+ = \imath(\phi_+ - \Theta_- j_+). \tag{62}$$

Remark that fields ϕ_+ introduced in (61) are the free fermions:

$$\partial_- \phi_+ = 0 \tag{63}$$

and with the conserved currents j_+ they generate (left-moving) N=1 super-Kac-Moody algebra of the model [28, 36]. It is easy to obtain from (58) PL T-duality transformation low for the currents (61)

$$t : \phi_+^\pm \to \breve{\phi}_+^\mp, \ j_+^\pm \to \breve{j}_+^\mp. \tag{64}$$

The generator of N=1 superconformal symmetry of the model is given by (we shall omit in the following the world sheet indeces $\pm$ having in mind that we are working in the left-moving sector)

$$\begin{aligned} \Gamma = &< DI, I > + \frac{1}{3} < \{I, I\}, I >= \\ < q, \phi > + \frac{\imath}{6} < \{\phi, \phi\}, \phi > &- \Theta_-(< q, q > - \imath < \partial\phi, \phi >) \equiv \Sigma + \Theta_- 2T. \end{aligned} \tag{65}$$

It can be represented as the sum

$$\begin{aligned} &\Gamma = \Gamma^+ + \Gamma^-, \\ \Gamma^\pm =< q^\pm, \phi^\mp > + \frac{\imath}{2} < \{\phi^\pm, \phi^\pm\}, \phi^\mp > &- \Theta_-(< q^\pm, q^\mp > - \imath < \partial\phi^\pm, \phi^\mp >). \end{aligned} \tag{66}$$

Thus we recognize spin 3/2- currents

$$\Sigma^\pm =< q^\pm, \phi^\mp > + \frac{\imath}{2} < \{\phi^\pm, \phi^\pm\}, \phi^\mp >, \tag{67}$$

stress-energy tensor

$$T = -\frac{1}{2}(< q, q > - \imath < \partial\phi, \phi >) \tag{68}$$

and $U(1)$ current

$$K =< \phi^+, \phi^- > \tag{69}$$

of the $N = 2$ super-Virasoro algebra so that

$$\Gamma^\pm = \Sigma^\pm + \Theta_-(T \pm \frac{\imath}{2}\partial K). \tag{70}$$

Using (61) we rewrite (70) in more convenient form

$$\Gamma^\pm =< j^\pm, \phi^\mp > - \imath < \{\phi^\mp, \phi^\mp\}, \phi^\pm > - \Theta_-(\frac{1}{2}(< j, j > - < \{\phi, \phi\}, j >) - \imath < \partial\phi^\pm, \phi^\mp >). \tag{71}$$

From (64) we obtain PL T-duality mapping for $\Gamma^\pm$:

$$t : \Gamma^\pm \to \check{\Gamma}^\mp, \tag{72}$$

or in components

$$t : \Sigma^\pm \to \check{\Sigma}^\mp, \; T \pm \imath\partial K \to \check{T} \mp \imath\partial\check{K}. \tag{73}$$

Thus PL T-duality in the $N = 2$ SWZNW models is a mirror duality. With the result of [24] that PL T-duality is some special super-Kac-Moody transformation it gives us more rigorous background for the conjecture proposed in [26].

Now we discuss Kazama-Suzuki models generalization of PL T-duality. Kazama and Suzuki have studied [25] under what conditions an N=1 superconformal coset model can have an extra supersymmetry, to give rise to an $N = 2$ superconformal model. Their conclusion can be formulated in terms of Manin triples: an $N = 2$ superconformal coset model is defined by a Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ and Manin subtriple $(\mathbf{h}, \mathbf{h}_+, \mathbf{h}_-)$, $\mathbf{h} \in$

$\mathbf{g}, \mathbf{h}_\pm \in \mathbf{g}_\pm$ so that the subspaces $\mathbf{l}_\pm \equiv \mathbf{g}_\pm/\mathbf{h}_\pm$ are the Lie algebras [40]. The spin-3/2 currents, stress-energy tensor and $U(1)$current of the $\mathbf{G}/\mathbf{H}$-coset model are given respectively by

$$\begin{aligned} \Sigma_c^\pm &=< q_l^\pm, \phi_l^\mp > + \frac{\imath}{2} < \{\phi_l^\pm, \phi_l^\pm\}, \phi_l^\mp >, \\ T_c &= -\frac{1}{2}(< q_l, q_l > - \imath < \partial\phi_l, \phi_l >), \\ K_c &=< \phi_l^+, \phi_l^- >, \end{aligned} \tag{74}$$

where the subscript l means the projection on the subspace $\mathbf{l} = \mathbf{g}/\mathbf{h}$.

Due to the denominator of the coset is $N = 2$ SWZNW model on the subgroup $\mathbf{H} \in \mathbf{G}$ PL T-duality maps the currents of this model by the formulas (64,73). Thus if we define PL T-duality transformation in $\mathbf{G}/\mathbf{H}$-Kazama Suzuki coset model as an appropriate projection of the transformation in $\mathbf{G}$-SWZNW model we obtain, taking into account (74), the formulas (73) as the transformation low of the $N = 2$ super-Virasoro algebra generators of the coset. Though in order to establish accurately PL T-duality in Kazama-Suzuki models an individual investigation should be done, we can conclude from these arguments that PL T-duality in the Kazama-Suzuki models is a mirror duality also.

It is pertinent to note that in contrast to the $N = 2$ SWZNW models the PL T-duality in Kazama-Suzuki models can be interpreted as a mirror symmetry between the Kähler manifolds. Indeed as it follows from the results of the paper [39] the nessesary condition for such interpretation is the realization of the (2,2) σ-model solely by chiral or solely by twisted chiral superfields. In this case the left-moving and right-moving spin-3/2 currents of the model are constructed with help of only one complex structure on the manifold so that we can associate the chiral-chiral and chiral-antichiral rings of the model with the De Ram and Dol'bo cohomology of a manifold [5]. Thus PL T-duality in the Kazama-Suzuki model can be related with the mirror symmetry of the Kähler manifolds if all primary fields of this model are only chiral or only twisted chiral fields. An intresting question which arises in this context is is there an analog of the $\mathbf{G}_+$- holomorphic action and PL symmetries in Calabi-Yau manifolds.

The discussion in this note was puerly classical. The natural and most important question what is the quantum picture of the PL T-duality in the $N = 2$ superconformal theories. Because the Poisson-Lie groups is nothing but a classical limit of the quantum groups [14] there appears an intriguing possibility of a relevance of quantum groups in the T-duality. In particular it would appear reasonable that mirror symmetry closely related with quantum group duality principle [41].

ACKNOWLEDGEMENTS

I am very grateful to A. Alekseev for helpful discussions and careful reading of the manuscript of the paper [24]. This work was supported in part by grants INTAS-95-IN-RU-690, CRDF RP1-277, RFBR 96-02-16507.

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